HARALD BOHR MEETS GEORGE BOOLE

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Harald Bohr's power series theorem states that for a holomorphic function f on the open unit disc $\mathbb{D}$ we have

$$\sum_{n=0}^{\infty} |f^{(n)}(0)| \, \frac{1}{3^n} \leq \sup_{z\in\mathbb{D}} |f(z)| \,,$$

and here the 3 can not be improved. This nowadays well-known result came out as a sort of by-product of Bohr's intensive study of ordinary Dirichlet series $\sum_n a_n n^{-s}$ from the beginning of the last century, and since then it remained a subject of special attention in various more general settings as e.g. for holomorphic functions on finite or infinite dimensional polydiscs.

After a short review of the historical background, we intend to report on a recent study of Bohr's phenomenon for real functions on the Boolean cube $\{-1,1\}^N$.

Every such function admits a canonical representation through its Fourier-Walsh expansion $f(x) = \sum_{S\subset\{1,\ldots,N\}} \widehat{f}(S)x^S$, where $x^S = \prod_{k\in S} x_k$. Given a class $\mathcal{F}$ of functions on the Boolean cube $\{-1,1\}^N$, the Boolean radius of $\mathcal{F}$ is defined to be the largest $\rho \geq 0$ such that $\sum_S |\widehat{f}(S)|\rho^{|S|} \leq \|f\|_\infty$ for every $f \in \mathcal{F}$. We indicate the precise asymptotic behaviour of the Boolean radius of several natural subclasses, as e.g. the class of all real functions on $\{-1,1\}^N$ or the subclass made of all homogeneous functions. Compared with the classical complex situation subtle differences as well as striking parallels occur, and moreover a somewhat curious link to the efficiency of quantum computers appears.

References

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